

Dates of Maximum of S Herculis, with New Elements. By the
Rev. T. E. R. Phillips, M.A. (Plate 19.)

The long-period variable S Herculis is of interest, apart from the nature of its light curve, inasmuch as the dates of maximum are seemingly irregular, and also appear to be affected by some periodic inequality. In his third Catalogue Chandler gave as the period of the star the simple value of 308.1 days with the note "large irregularities," but in his revised Catalogue a sine term was added, the formula being

$$2399197 + 308.3E + 35 \sin(9^\circ E + 86^\circ).$$

In vol. lv. of the *Memoirs* of the Society Professor Turner compared the Rousdon observations with Chandler's revised formulæ, and pointed out that, although the new formulæ were generally found to be a great improvement on the old, the case of S Herculis was a striking exception.* Later the observations of this star by the members of the Variable Star Section of the British Astronomical Association were published (*B.A.A. Memoirs*, vols. xv. and xviii.), and subsequently the individual light curves based on the work of the Section down to the end of 1909 (Appendix to vols. xv. and xviii.). This data has been further supplemented through the kindness of Mr. C. L. Brook, the present Director of the Section, in supplying privately the unpublished means of the Section's observations of S Herculis from 1910 to the end of 1914. Quite recently the editing and publishing of Baxendell's observations by Professor Turner and Miss Blagg (*M.N.*, vol. lxxv., No. 5, p. 398) has provided additional material of an early date. Baxendell's observations are in two groups separated by an interval of about nine years. This interval is to be regretted as, apart from this, we now have an almost complete record from 1857 to the present time. At the conclusion of their paper Professor Turner and Miss Blagg tabulate a number of solutions for the dates of maximum, which, however, were worked out without making use of the B.A.A. results. The present paper is an attempt to find a solution which will satisfy the whole of the material now available.

1. In a discussion of this sort it seems desirable to consider first what is to be taken as the date of maximum of a variable star, for there are two possible alternatives: (1) the date on which the star appears to have been actually brightest; (2) the date indicated by considerations of symmetry with regard to the star's rise and decline as determined by the gradient at other parts of the curve.

* The error of Chandler's formula is now ($E=70$), about +120 days. Guthnick's formula, given in Hartwig's *Ephemeris*, is:— $2399219 + 307.5E + 47 \sin(7^\circ.5E + 120^\circ) + 12 \sin(15^\circ.0E + 60^\circ)$, and its present error about +50 days.

The former and perhaps the more natural plan seems open to the objections: (a) that the highest point on a light curve is often very difficult to determine (see remarks by Professor Turner, *M.N.*, vol. lxvii., No. 5, p. 337 *et seq.*), and (b) that observations near maximum are frequently liable to error owing to the star's redness or the want of suitable comparison stars in the field. The alternative plan (2) has the advantages (a) of representing the curve more or less as a whole, and (b) that, provided a sufficient portion of the curve is available, the probable date of maximum (compared with the adopted maximum of the mean curve) can be computed when the star was unobserved or unobservable about that time. The formula for correction to epoch given by Professor Turner in *M.N.*, vol. lxxiii., No. 2, p. 134, is

$$\frac{N}{2\pi} \Sigma D(p-f) / \Sigma (p-f)^2,$$

where N = the adopted period of the star, D the residual at any point referred to the mean light curve at that point, $p-f$ = the difference between the values of the ordinates preceding and following the ordinate in question, as read from the mean curve.

In Table I. are brought together and compared the dates of maximum obtained by both methods. Column 1 gives the epoch or number of the period; column 2, the "observed" dates of maximum, or the dates as read from the plotted curves in the neighbourhood of maximum; column 3, the dates of maximum computed by the formula above given. In column 2 the dates of epochs 1 to 34 are estimated from Baxendell's observations published by Professor Turner and Miss Blagg (*M.N.*, vol. lxxv., No. 5, p. 413 *et seq.*); those of epochs 37 to 51 are similarly estimated from the Rousdon observations in vol. lv., *R.A.S. Memoirs*, p. 90 *et seq.*, and the remaining dates are obtained from the light curves drawn from the B.A.A. Variable Star Section's observations. In column 3 the dates of epochs 1 to 34 are taken from the paper by Professor Turner and Miss Blagg (vol. lxxv., No. 5, p. 420); those of epochs 37 to 52 have been computed from the Rousdon residuals published in vol. lv. of the *Memoirs* of the Society, pp. lxxxv and lxxxvi, and those of epochs 58 to 69 are computed from the observations of the B.A.A. Variable Star Section. Column 4 gives the difference between the two sets of dates, and it will be seen that although there is a general agreement which one would expect, there are sometimes discrepancies amounting to nearly 20 days.

TABLE I.

Dates of Maximum of S Herculis.

E.	"Observed" Dates. J.D.	Computed Dates. J.D.	O - C.	E.	"Observed" Dates. J.D.	Computed Dates. J.D.	O - C.
1	2399565	2399568.9	- 3.9	43	...	2477.4	...
2	9879	(9876.9)	+ 2.1	44	2780	(2789.7)	- 9.7
3	2400183	2400177.8	+ 5.2	45	3098	(3099.4)	- 1.4
6	...	1077.9	...	46	3408	3405.4	+ 2.6
7	1379	1376.4	+ 2.6	47	3720	3715.0	+ 5.0
8	1684	1678.5	+ 5.5	48	...	(4023.2)	...
9	...	(1983.0)	...	49	...	4332.4	...
10	...	(2279.7)	...	50	4631	(4611.9)	+ 19.1
11	...	(2572.0)	...	51	4922	4922.2	- 0.2
12	2877	2874.9	+ 2.1	52	5238	5241.1	- 3.1
13	3185	(3179.8)	+ 5.2	53	5542
14	3476	56	6433
.	.	.	.	57	6738
26	(7212)	(7208.0)	+ 4.0	58	7049	7049.4	- 0.4
27	...	(7521.0)	...	59	7345	7344.0	+ 1.0
28	7826	60	7661	7649.7	+ 11.3
29	...	8137.6	...	61	7963	7954.7	+ 8.3
30	...	(8456.4)	...	62	8250	8252.9	- 2.9
33	(9383)	9394.9	- 11.9	63	8562	8560.6	+ 1.4
34	9701	9707.5	- 6.5	64	(8867)	8864.3	+ 2.7
37	2410615	(2410622.9)	- 7.9	65	9155	9157.1	- 2.1
38	0925	0926.5	- 1.5	66	(9433)	9451.5	- 18.5
39	1239	1227.6	+ 11.4	67	9761	9753.8	+ 7.2
40	1560	1541.7	+ 18.3	68	2420047	2420055.6	- 8.6
41	(1865)	(1847.3)	+ 17.7	69	0350	0357.1	- 7.1
42	...	2162.9	...				

N.B.—The figures in brackets are somewhat uncertain owing to the paucity of observations or other reasons.

2. An alternative method of discussion would be to make use of the dates of minimum, but the curve is usually rather flat near that point, and the faintness of the star at such times placed it beyond the range of Baxendell's instruments.

3. The question now arises as to the homogeneity of the above material. Are the different series of observations mutually comparable, or are they appreciably affected by some kind of personal equation? Unfortunately we have no means of directly comparing

Baxendell's with the Rousdon series, but the Rousdon observer,* Mr. C. Grover, has very kindly forwarded privately his observed dates of maximum since those published in vol. lv. of the *Memoirs*, and these can be compared with those taken from the light curves of the B.A.A. Variable Star Section. The result of the comparison is shown in Table II.

TABLE II.
"Observed" Dates of Maximum of *S Herculis*.

E.	Rousdon. J.D.	B.A.A. J.D.	Rousdon, B.A.A.	E.	Rousdon. J.D.	B.A.A. J.D.	Rousdon, B.A.A.
52	2415238	2415238	0	63	8550	8562	- 12
53	5521	5542	- 21	64	8865	(8867)	- 2
57	6746	6738	+ 8	67	9739	9761	- 22
58	7035	7049	- 14	68	2420029	2420047	- 18
59	7345	7345	0	69	0325	0350	- 25
62	8236	8250	- 14				

It will be seen that the differences, especially towards the end of the series, are sometimes very considerable. The idea suggests itself that possibly there is a change of colour in the star at or after its rise to maximum. To an inquiry on this point Mr. Grover replied that, as is the case with many other long-period variables, the red colour is palest at maximum and deepens as the light wanes. When about 9 m. or less it is often of a very deep ruddy tone. There are, however, variations from the general rule, *e.g.* Mr. Grover says that in 1887, 1888, and 1889 the star was fiery red at maximum, and in 1902 deep orange. In other years it was pale yellow or almost white. Reference may be made here to the case of T Cephei, discussed by Professor Turner in vol. lv., *Memoirs*, p. civ, and *Memoirs*, B.A.A., vol. xviii. p. 6 *et seq.*, from which it is abundantly clear that under certain circumstances observers may differ among themselves at the same magnitudes according as the star is on the rise or fall. This may be due to some difference of colour at the two parts of the curve; and S Herculis may present an analogous case. The question of the telescope used—whether a reflector or refractor, and if the latter, the nature of its colour correction—should also be considered, but there is not the material for a discussion of the point here.

4. The question must now be settled as to how the dates in Table I. are to be treated in the final analysis. Are columns 2 and 3 to be somehow combined, or is one of them to be chosen in preference to the other, and if so which? On consideration a method of compromise seemed unsatisfactory, and it is perhaps

* After this was written I received from Mr. Grover a diagram of his observations of the star in 1886, together with those of Baxendell. These were Mr. Grover's first and Baxendell's last observations of this star. The accordance is satisfactory, but the observations of Baxendell are too few and scattered to establish any comparison between the habit of the two observers.

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desirable to find a solution for each series separately. But there still remains the question of the discrepancy between Rousdon and the B.A.A. Variable Star Section. The mean value of Rousdon—B.A.A. is -11 , but this is largely due to the last three residuals at $E=67, 68$, and 69 . Neglecting these, the addition of 6 days to the observed Rousdon dates will practically bring them into line with the B.A.A. series, and on the whole it seems preferable to take the latter as the standard since they are based upon the work not of one but of a number of observers.

As regards the computed dates in column 3 of Table I. it is not at first so clear that the same correction of $+6$ days should be applied, for it might be supposed that the discrepancy is a peculiarity connected with the star's appearance at maximum. To test this point the mean light curves of the Baxendell, Rousdon, and B.A.A. series are brought together as follows:—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Baxendell	11.7	(11.9)	11.3	10.2	9.3	8.6	8.0	7.7	8.3	9.2	10.2	11.0
Rousdon	12.0	11.8	11.0	10.3	9.7	8.6	7.9	7.6	8.1	9.1	10.1	11.4
B.A.A.	12.0	12.0	11.0	9.8	8.8	8.1	7.6	7.7	8.8	9.1	10.9	11.7

Now on plotting these it is found that there is no evidence whatever of an accelerated Rousdon maximum as compared with the other two curves. It is accordingly concluded (taking the B.A.A. curve as standard) that in the observations the Rousdon curve is shifted bodily, and that the correction $+6$ days must be applied to the computed as well as to the "observed" dates to make the material homogeneous.

6. Turning now to the analysis of the dates, the first question is that of the mean period. The periods for Baxendell's two series ($E=1$ to 13 , and $E=26$ to 34), as given by Professor Turner and Miss Blagg (*M.N.*, vol. lxxv. pp. 420 and 422), are 300.1 and 312.5 days respectively. Professor Turner found for the Rousdon series at $E=41$, $P=310.3$ days; and at $E=46$, $P=307.6$ days, or 309 days in the mean. The B.A.A. observations give for $E=58$ to 61 , $P=302.7$ days; for $E=62$ to 65 , $P=300.5$ days; and for $E=66$ to 69 , $P=299.5$ days. The value 300.1 days given by Professor Turner and Miss Blagg applies to Baxendell's first series as a whole; but making use of the available residuals on p. 419 of *M.N.*, vol. lxxv., No. 5, we get for $E=1$ to 7 , $P=300.6$ days, and for $E=7$ to 14 , $P=298.4$ days. These figures are perhaps not very reliable owing to the scantiness of material available for such a determination, but on the whole it seems likely that the period of the star was about the same at $E=3$ and $E=64$. Taking, then, the available pairs of observations at intervals of 61 epochs, we get from both series of maxima a mean value of 306.2 days, which is probably not far from the truth.

7. In Table III. the material for analysis is assembled and compared with a uniform period of 306.2 days; but the correction

of +6 days, as found above, has been applied to the Rousdon figures in columns 2 and 3 of Table I. for E=37 to 51, and E=37 to 52 respectively.

TABLE III.

E.	"Observed" Date. J.D.	Computed Date. J.D.	Date from Uniform Period of 306'2 days. (2399260'8+306'2 E.)	O - Uniform Period.	C - Uniform Period.
1	2399565	2399568'9	2399567'0	- 2'0	+ 1'9
2	9879	(9876'9)	9873'2	+ 5'8	+ 3'7
3	2400183	2400177'8	2400179'2	+ 3'8	- 1'4
6	...	1077'9	1098'0	...	- 20'1
7	1379	1376'4	1404'2	- 25'2	- 27'8
8	1684	1678'5	1710'4	- 26'4	- 31'9
9	...	(1983'0)	2016'6	...	- 33'6
10	...	(2279'7)	2322'8	...	- 43'1
11	...	(2572'0)	2629'0	...	- 57'1
12	2877	2874'9	2935'2	- 58'2	- 60'3
13	3185	(3179'8)	3241'4	- 56'4	- 61'6
14	3476	...	3547'6	- 71'6	...
26	(7212)	(7208'0)	7222'0	- 10'0	- 14'0
27	...	(7521'0)	7528'2	...	- 7'2
28	7826	...	7834'4	- 8'4	...
29	...	8137'6	8140'6	...	- 3'0
30	...	(8456'4)	8446'8	...	+ 9'6
33	(9383)	9394'9	9365'4	+ 17'6	+ 29'5
34	9701	9707'5	9671'6	+ 29'4	+ 35'9
37	2410621	(2410628'9)	2410590'2	+ 30'8	+ 38'7
38	0931	(0932'5)	0896'4	+ 34'6	+ 36'1
39	1245	1233'6	1202'6	+ 42'4	+ 31'0
40	1566	1547'7	1508'8	+ 57'2	+ 38'9
41	(1871)	(1853'3)	1815'0	+ 56'0	+ 38'3
42	...	2168'9	2121'2	...	+ 47'7
43	...	2483'4	2427'4	...	+ 56'0
44	2786	(2795'7)	2733'6	+ 52'4	+ 62'1
45	3104	(3105'4)	3039'8	+ 64'2	+ 65'6
46	3414	3411'4	3346'0	+ 68'0	+ 65'4
47	3726	3721'0	3652'2	+ 73'8	+ 68'8
48	...	(4029'2)	3958'4	...	+ 70'8
49	...	4338'4	4264'6	...	+ 73'8
50	4637	(4617'9)	4570'8	+ 66'2	+ 47'1
51	4928	4928'2	4877'0	+ 51'0	+ 51'2

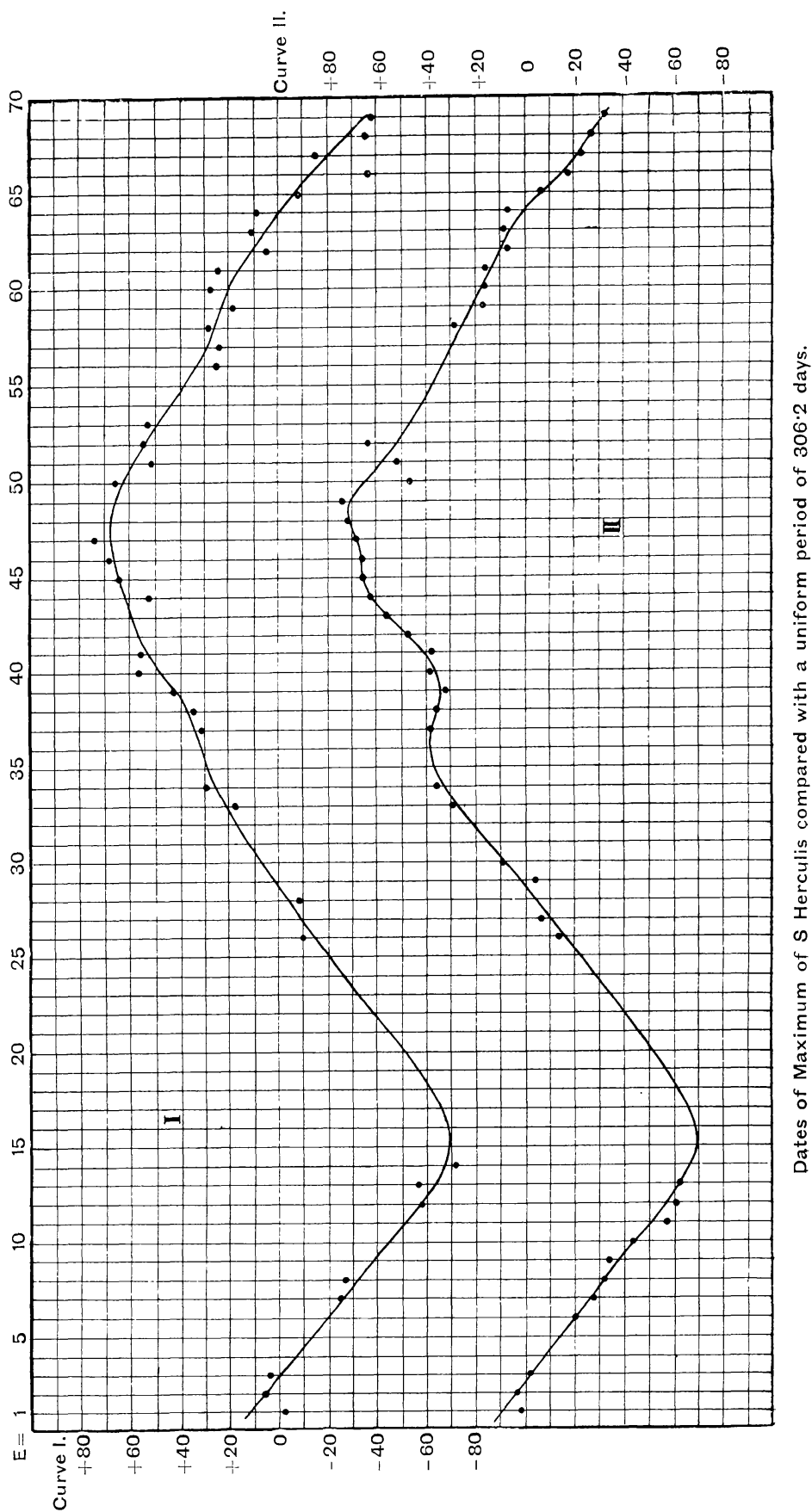
TABLE III.—*continued.*

E.	"Observed" Date. J.D.	Computed Date. J.D.	Date from Uniform Period of 306·2 days. (2399260·8+306·2 E.)	O—Uniform Period.	C—Uniform Period.
52	2415238	2415247·1	2415183·2	+54·8	+63·9
53	5542	...	5489·4	+52·6	...
56	6433	...	6408·0	+25·0	...
57	6738	...	6714·2	+23·8	...
58	7049	7049·4	7020·4	+28·6	+29·0
59	7345	7344·0	7326·6	+18·4	+17·4
60	7661	7649·7	7632·8	+28·2	+16·9
61	7963	7954·7	7939·0	+24·0	+15·7
62	8250	8252·9	8245·2	+4·8	+7·7
63	8562	8560·6	8551·4	+10·6	+9·2
64	(8867)	8864·3	8857·6	+9·4	+6·7
65	9155	9157·1	9163·8	—8·8	—6·7
66	(9433)	9451·5	9470·0	—37·0	—18·5
67	9761	9753·8	9776·2	—15·2	—22·4
68	2420047	2420055·6	2420082·4	—35·4	—26·8
69	0350	0357·1	0388·6	—38·6	—31·5

8. Plotting the residuals in columns 5 and 6 of the above table we obtain the curves I. and II. (see Plate 19), in regard to which the following points seem worth noticing:—

- (a) The periodic inequality, which is often masked by minor irregularities when only a few periods are considered, is established beyond question. This is of interest in view of the doubt which has sometimes been cast on such alleged inequalities.
- (b) The inequality is obviously not a simple sine term, but is of considerable complexity.
- (c) Accidental errors, if we may so call them, are greatly reduced when the dates of maximum are calculated from the curve. Save for the apparent discrepancies in curve II. at E=50, 51, and 52, which may very possibly be due to a real kink at that part, the dates are seen to be very consistent, though the curve on which they lie is complicated.
- (d) Curve II. exhibits a well-marked hump and depression—slightly indicated in curve I.—which resembles the pause on the rise shown by the star's light curve.

9. In view of the doubt attached to certain parts of the curves, *e.g.* the gap in Baxendell's observations and the neighbourhood of E=50 to 57, it will be sufficient for the analysis to take the readings at twelve equidistant ordinates within a complete period of the inequality. We then get the figures in Table IV.



Dates of Maximum of S Herculis compared with a uniform period of 306.2 days.

TABLE IV.

Ordinates of Curves I. and II.

E.	Curve I.	Curve II.	E.	Curve I.	Curve II.
3	0	- 2	33½	+ 23	+ 30
8½	- 32	- 32	38½	+ 40	+ 34
13½	- 64	- 63	43½	+ 61	+ 60
18½	- 60	- 60	48½	+ 68	+ 71
23½	- 31	- 32	53½	+ 45	+ 43
28½	- 2	- 1	58½	+ 24	+ 22

Assuming these curves to be of the form—

$A + a \sin \theta + b \cos \theta + c \sin 2\theta + d \cos 2\theta + e \sin 3\theta + f \cos 3\theta$, etc.,
and analysing them harmonically we get:

Curve I. $A - 58.5 \sin \theta - 14.7 \cos \theta - 4.3 \sin 2\theta + 3.8 \cos 2\theta + 5.0 \sin 3\theta + 4.3 \cos 3\theta$.

Curve II. $A - 58.0 \sin \theta - 15.5 \cos \theta - 4.8 \sin 2\theta + 4.0 \cos 2\theta + 7.0 \sin 3\theta + 2.7 \cos 3\theta - 0.2 \sin 4\theta + 4.0 \cos 4\theta - 0.7 \sin 5\theta - 3.3 \cos 5\theta$.

These curves may be put in the form—

Curve I. $A + 60.3 \sin (\theta + 194^\circ.1) + 5.7 \sin (2\theta + 138^\circ.5) + 6.6 \sin (3\theta + 40^\circ.7)$.

Curve II. $A + 60.0 \sin (\theta + 195^\circ.0) + 6.1 \sin (2\theta + 140^\circ.2) + 7.5 \sin (3\theta + 21^\circ.1) + 4.0 \sin (4\theta + 87^\circ.1) + 3.4 \sin (5\theta + 258^\circ.0)$.

It will be seen that the first three harmonics in the two curves are very similar. The fourth and fifth harmonics are insignificant in curve I. and are omitted, but have a considerable amplitude in curve II.

10. We may now give the final solutions for the dates of maxima of S Herculis. Taking the means of columns 2 and 3 of Table IV. we get the values +6 and +5.7. We therefore add these to the constant at the head of column 4 of Table III., which becomes 2399266.8 and 2399266.5. Since the inequality runs through its course in 61 epochs, the argument will have the value $5^\circ.9$. The analyses above given are for epochs 3 to 64, and the value of 3 epochs expressed as an angle is $17^\circ.7$. Correcting, then, the curves to $E=0$ we have the following as the two formulæ for the star:—

I. $2399266^d.8 + 306^d.2 E + 60^d.3 \sin (5^\circ.9 E + 176^\circ.4) + 5^d.7 \sin (11^\circ.8 E + 103^\circ.1) + 6^d.6 \sin (17^\circ.7 E + 347^\circ.6)$.

II. $2399266^d.5 + 306^d.2 E + 60^d.0 \sin (5^\circ.9 E + 177^\circ.3) + 6^d.1 \sin (11^\circ.8 E + 104^\circ.8) + 7^d.5 \sin (17^\circ.7 E + 328^\circ.0) + 4^d.0 \sin (23^\circ.6 E + 16^\circ.3) + 3^d.4 \sin (29^\circ.5 E + 169^\circ.5)$.

11. As a test of the analysis it will suffice to take curve II. Comparing the dates in column 3 of Table III. with those com-

puted by the second formula, we obtain the residuals shown in Table V.

TABLE V.

E.	Adopted Date - Formula. d	E.	Adopted Date - Formula. d	E	Adopted Date - Formula. d
1	- 5.2	33	+ 1.6	50	(- 19.2)
2	(+ 1.2)	34	+ 4.8	51	- 9.4
3	+ 1.2	37	(+ 6.3)	52	+ 9.4
6	- 1.8	38	(+ 2.0)	58	+ 4.0
7	- 3.6	39	- 4.3	59	- 3.6
8	- 1.4	40	+ 0.8	60	+ 0.1
9	(+ 3.9)	41	(- 4.1)	61	+ 3.4
10	(+ 1.5)	42	- 0.4	62	+ 0.2
11	(- 5.3)	43	+ 1.3	63	+ 6.6
12	- 2.2	44	(+ 0.8)	64	+ 9.2
13	(+ 1.8)	45	(- 1.3)	65	+ 0.8
26	(+ 3.9)	46	- 5.7	66	- 5.8
27	(+ 4.8)	47	- 4.9	67	- 4.2
29	- 5.4	48	(- 2.4)	68	- 2.7
30	(+ 0.1)	49	+ 3.2	69	- 1.0

It will be seen that with few exceptions these residuals are reasonably small, and probably within the limits of error in the deduced dates of maximum.

We may also compare the corrected periods found for the Baxendell, Rousdon, and B.A.A. series with those given for the same epochs by the formula (II.) This is done in Table VI.

TABLE VI.

E.	Periods.		O - F.
	From Observations.	From Formula.	
	d	d	d
1-7	300.6	301.0	- 0.4
8-13	298.4	299.6	- 1.2
26-34	312.5	312.3	+ 0.2
39-43	310.3	311.0	- 0.7
44-49	307.6	308.1	- 0.5
58-61	302.7	302.0	+ 0.7
62-65	300.5	301.2	- 0.7
65-69	299.5	300.3	- 0.8

Here, again, the residuals are satisfactory. It cannot be claimed, however, that the formula is definitive for the following reasons :—

- (a) Uncertainty as to the true mean value, which may be more or less than 306.2 days ;
- (b) Doubt as to the homogeneity of the material used in the discussion, and as to the true form of certain parts of the curve ;
- (c) The probability that, as is the case with the individual light curves, the curve is not exactly reproduced in successive periods of the inequality.

It may be remarked that the curve of dates should reach a minimum, *i.e.* the period of the star should be at its mean value a few years hence, and future observations will be of interest as a test of the work here tabulated.

12. Some acknowledgment must be made, finally, of the work of those who have provided the data for a discussion of this star.

It is a cause for great satisfaction that Professor Turner and Miss Blagg should be publishing in a form convenient for use the observations made so long ago by Baxendell, without which in some cases such discussions would be impossible.

Mr. Grover's persistent work at the Rousdon Observatory has not only carried on the observation of the star from the point where Baxendell left it, but its continuance to the present time adds enormously to its value ; while the Variable Star Section of the British Astronomical Association, by combining the work of so many observers, is contributing material of inestimable worth to future investigators of stellar variation.

Ashted :
1915 May 8.

On the Short-Period Variable RR Lyræ. By C. Martin and
H. C. Plummer. (Plates 20, 21.)

1. RR Lyræ is a variable of the cluster type discovered by Mrs. Fleming in 1901. Its designation in the Harvard Catalogue of variables is 192242, and it is identical with B.D. + 42° 3338 and with A.G.C. (Bonn) 12959. One hundred and eight photographic observations have been secured by Mr. Martin with the 15-inch reflector of this observatory between 1913 October and 1914 November. All the plates have been measured and reduced by him. The method of reduction has been sufficiently indicated in a series of papers on similar variables recently published by us in the *Monthly Notices*.

2. The magnitudes of the comparison stars have been based, as before, on two plates exposed under similar conditions both on the region of the variable and on the χ Persei cluster. The